Journal of Engineering Physics and Thermophysics, Vol. 80, No. 2, 2007

CALCULATION OF RADIATION FLUXES TO THE SURFACE OF A SPACE VEHICLE WITH THE AID OF A DISCRETE ORDINATES METHOD

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A method and results of numerical modeling of radiative heating of the back surface of the MSRO (Mars Sample Return Orbiter) space vehicle of the European Space Agency are presented. To determine radiation heat fluxes, the method of discrete ordinates on unstructured tetrahedral grids is used. The radiative model is based on the radiation-transfer equation in a multigroup approximation. Numerical calculation has been performed for the most thermally stressed point of the assumed trajectory of the entry of an MSRO-type space vehicle into the Mars atmosphere. A comparison with the discrete ordinates method on structured grids is made. Good agreement between the results of calculations on structured and unstructured grids is demonstrated. The level of radiation heat fluxes to the back surface of the MSRO space vehicle is predicted.

Introduction. The use of space vehicles to study the Solar system planets attracts practical and theoretical interest in the study of the processes proceeding on entry of descent modules into the dense layers of the atmospheres of planets. At velocities amounting to several thousands of kilometers per second, a space vehicle is subjected to strong convective and radiative heating as a result of interaction with a gas.

Under these conditions high temperatures arise behind a shock-wave front. In order to reduce the overall weight of a space vehicle, it is necessary to precisely calculate its thermal shielding. For this purpose, it is important to calculate, with a high reliability, the heat fluxes onto the surface of a landing module. Estimation of the convective and radiative heating is usually made for a forward aerodynamic shield which takes the main part of the heat load. However, during the entry of a space vehicle into the Mars atmosphere the radiative heating of its back surface may also play an important role, since this surface is subjected to the action of the radiation heat flux emitted by tens of cubic meters of carbon dioxide heated to high temperatures, which is known to be a good emitter in the infrared spectral range. Thus, one of the problems of aerothermodynamic analysis of descent modules is the prediction of the intensity of radiative heating of their back surfaces.

The discrete ordinates method (DOM) has been much studied and has long been in use for calculating thermal radiation transfer. Recent advances in its development are associated with numerical calculations in an axisymmetric cylindrical statement [1] and with integration of the transfer equation in a rectangular Cartesian coordinate system in a three-dimensional statement [2]. In [3], a modification of the DOM is presented in a three-dimensional geometry in application to media enclosed in volumes with locally heated walls. In [4], the DOM was used to find the spectral characteristics of the thermal radiation field of the plasma of an axisymmetric arc discharge in an argon atmosphere. Radiative heating of the inner surface of air and hydrogen laser plasma generators with the use of DOM was investigated in [5, 6].

The DOM is not the only acceptable method for solving the problem posed. In [7], an analysis of other methods is given: the method of spherical harmonics, the ray-tracing method, and Monte Carlo simulation. Among them, the DOM has a number of advantages (computational economy, precision), which would be appropriate for use in solving the problem posed. However, there are a number of unresolved problems impeding the application of the method. The main among them is the difficulty of using the DOM to solve the equation of thermal radiation transfer in arbitrary volumes of complex geometry.

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 1062-0125/07/8002-0284

In [8], a modification of the standard DOM is suggested, which allows one to solve the radiation transfer equation on unstructured tetrahedral grids. The use of DOM on such grids makes it possible to considerably extend the range of solvable problems in the theory of thermal radiation transfer, eliminating restrictions on the geometry used. In the present work, the algorithm suggested in [8] is being developed, which is employed to find radiation fluxes to the surface of a space vehicle of the MSRO type of the European Space Agency.

Formulation of the Method. The DOM in an arbitrary three-dimensional geometry is formulated for a computational domain, in which an unstructured grid containing a finite number of nonoverlapping tetrahedrons is introduced. The radiation transfer equation in the DOM representation has the form

$$\eta_m \frac{\partial I_\lambda^m}{\partial x} + \mu_m \frac{\partial I_\lambda^m}{\partial y} + \xi_m \frac{\partial I_\lambda^m}{\partial z} = \kappa_\lambda \left(I_{\mathrm{b},\lambda} - I_\lambda^m \right), \tag{1}$$

where I_λ^m is the spectral radiation intensity dependent on the space coordinates x, y, z and the direction $\mathbf{\Omega}_m$; $I_{\mathrm{b},\lambda}$ is the spectral intensity of the radiation of a blackbody at the temperature of the medium; and κ_λ is the spectral absorption coefficient of the medium.

Specification of radiation $I_{\lambda,\Gamma}^m$ at the boundary of the computational volume Γ allows one to formulate the boundary condition of the form

$$I_\lambda^m = I_{\lambda,\Gamma}^m \quad \text{for} \quad \left(\mathbf{n}_\Gamma \cdot \mathbf{\Omega}_m \right) > 0 \,, \tag{2}$$

where $\mathbf{n}_\Gamma$ is the normal to the boundary of Γ directed inside the computational domain.

We will replace the spectral quantities by group ones and integrate Eq. (1) over the volume of a tetrahedral cell:

$$\sum_{i=1}^{4} \left(\mathbf{n}_i \cdot \mathbf{\Omega}_m \right) S_i \, I_i^m = \kappa_p V_p \left(I_{\mathrm{b}p} - I_p^m \right), \tag{3}$$

where $I_i^m = \frac{1}{S_i} \int\limits_{S_i} I^m dS$ is the averaged group intensity over the area of the ith face of the tetrahedral cell; $I_p^m = \frac{1}{V_p} \int I^m dV$ is the mean group intensity in the volume of the cell; $\mathbf{n}_i$ is the vector of the normal to the face numbered i; κ_p is the group absorption coefficient in the cell; $I_{\mathrm{b}p}$ is the group radiation of a blackbody at the temperature of the medium in the cell numbered p. The group characteristics are determined by the formula $f = \frac{1}{\Delta\lambda} \int\limits_{\Delta\lambda} f_\lambda d\lambda$, where f_λ is the spectral quantity, f is the group quantity, and $\Delta\lambda$ is the group range of averaging.

Representing the group intensity at the center of the cell from Eq. (3) in terms of the mean group intensities on the faces, we obtain

$$I_p^m = I_{\mathrm{b}p} - \frac{1}{\kappa_p V_p} \sum_{i=1}^{4} \left(\mathbf{n}_i \cdot \mathbf{\Omega}_m \right) S_i \, I_i^m \,. \tag{4}$$

In order to solve Eq. (4), it is necessary to formulate the equation of coupling between the mean intensities on the faces of the tetrahedral cell. Three possible cases of the propagation of radiation inside the cell are presented in Fig. 1. If one face receives radiation from the remaining three faces (Fig. 1a), the mean intensity on receiving face 4 depends on the intensities of the remaining faces as follows:

$$I_4^m = \left(\frac{S_{14}}{S_4} I_1^m + \frac{S_{24}}{S_4} I_2^m + \frac{S_{34}}{S_4} I_3^m \right) \chi_p + I_{\mathrm{b}p} \, (1 - \chi_p) \,. \tag{5}$$

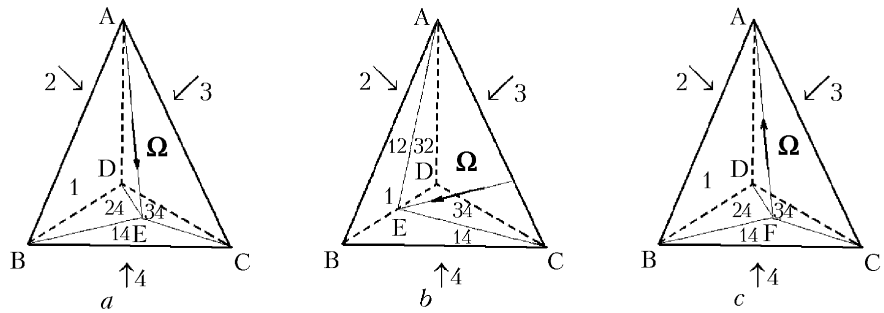

Fig. 1. Three possible versions of radiation propagation in a tetrahedral cells: a) one face receives radiation from the remaining three; b) two faces receive radiation from other two; c) three faces receive radiation from the fourth one.

In the second case (Fig. 1b), where two faces receive radiation from the remaining two ones, the intensity on the receiving faces is

$$I_2^m = \left(\frac{S_{12}}{S_2} I_1^m + \frac{S_{32}}{S_2} I_3^m \right) \chi_p + I_{\mathrm{b}p} \, (1 - \chi_p) \, , \tag{6}$$

$$I_4^m = \left(\frac{S_{14}}{S_4} I_1^m + \frac{S_{34}}{S_4} I_3^m \right) \chi_p + I_{\mathrm{b}p} \, (1 - \chi_p) \, . \tag{7}$$

In Eqs. (5)–(7), the areas with double subscripts are used. Figure 1 illustrates the origin of these subscripts. For example, S_{24} means the area of the projection of face numbered 2 onto the face numbered 4 formed by rays propagating in the direction $\mathbf{\Omega}_m$.

The third case (Fig. 1c) describes the situation where the three faces receive radiation from the fourth one. The intensities on the faces that receive radiation from the face numbered 4 are

$$I_1^m = I_2^m = I_3^m = I_4^m \chi_p + I_{\mathrm{b}p} \, (1 - \chi_p) \, . \tag{8}$$

Equations (5)–(8) involve the weight function χ_p, which can be represented as

$$\chi_p = \frac{2}{\tau_p} \left(1 - \frac{1 - \exp \, (- \, \tau_p)}{\tau_p} \right), \tag{9}$$

where τ_p is the maximum optical length in the cell p in the direction of radiation propagation $\mathbf{\Omega}_m$.

Computational Grids. Since calculation of radiation transfer is closely related to the problem of finding gas-dynamic functions, it is often advisable to use the same computational grids. Most common for three-dimensional problems are unstructured tetrahedral grids.

In solving a three-dimensional problem, a three-dimensional hexahedral cell is divided into six tetrahedrons ABCF, AFDE, FCDA, DEGH, CEFD, and GFCD (Fig. 2a). If the region has an axial symmetry, the grid in a cylindrical coordinate system contains three-dimensional pentahedral cells bordering on the z axis; they are divided into three tetrahedrons ABCD, BCEF, and CDEB (Fig. 2b). Figure 3a presents the cross section of a tetrahedral grid obtained as a result of the division of the cylindrical grid into tetrahedrons. If the statement of the problem presupposes axisymmetric specification of the parameters of the medium, then it is sometimes preferable to use quasi-uniform distribution of cells in a cylindrical volume. An example of the cross section of a quasi-uniform grid is presented in Fig. 3b.

Along with the spatial discretization, a grid in the direction of a round solid angle is also introduced. In the presence of the process of scattering, in the equation of thermal radiation transfer the integral of scattering over the

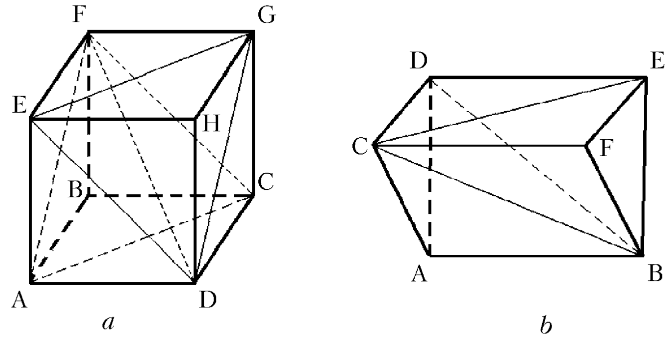

Fig. 2. Division into tetrahedrons of a hexahedral cell (a) and of the cell bordering on the axis z (b).

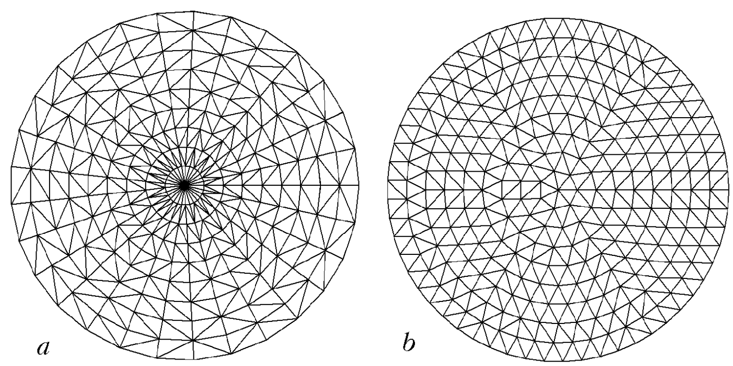

Fig. 3. Cross section of a cylindrical grid divided into tetrahedrons (a) and of a cylindrical volume with a quasi-uniform grid (b).

solid angle in the DOM is replaced by quadrature expansion in discrete directions [9]. The accuracy of approximation of the scattering integral is determined by the accuracy with which the integral is expanded into quadrature formulas. The thermal radiation transfer equation is solved for each individual angular direction. In the absence of scattering these directions can be selected arbitrarily. However, in finding the thermal radiation field characteristics (for example, thermal radiation fluxes) it is necessary to carry out integration over the angle. These integrals, in turn, can be calculated with the aid of expansion in quadrature formulas. Therefore, it is worthwhile to introduce an angular grid that coincides with the directions of such a formula. It has been proved in the DOM theory that to raise the accuracy of calculations, it is preferable that the quadrature scheme selected make it possible to find, with the least error, the first moments and be invariant relative to rotation through 90^{o} around the principal coordinate axes [9]. In the present work the S_N quadrature is used which satisfies the above-listed requirements. The nodes of such a quadrature are located on the sphere at the points of intersection of latitudes. The value of N specifies the number of projections of directions obtained for each principal coordinate axis. Precisely this N is meant when the order of the S_N quadrature is mentioned. The number of discrete directions in the S_N quadrature is defined by the formula $N(N+2)$ and depends on the order of N.

Order of Computations. In order to calculate the mean intensity in the volume of a cell with the aid of Eq. (4) it is necessary to know the intensities on the faces. To solve the problem of radiation transfer, an algorithm for recognition of the spatial mutual orientation of the cell and direction of radiation propagation $\mathbf{\Omega}_m$ was developed. This algorithm allows one to select characteristic equations (5)–(8) for the case corresponding to the variant of radiation propagation in the cell. If the intensity on the emitting faces is known, then the algorithm written as a computer code calculation of intensities of the receiving faces is used. The mean volumetric intensity at the center of the cell can be determined with the aid of Eq. (4). Calculation in the entire three-dimensional region is carried out successively.

When unstructured grids are used, the sequence of the passage of cells for each direction selected is not evident. Therefore, to reduce the expenditures of machine time it is useful, before the start of computation, to create a

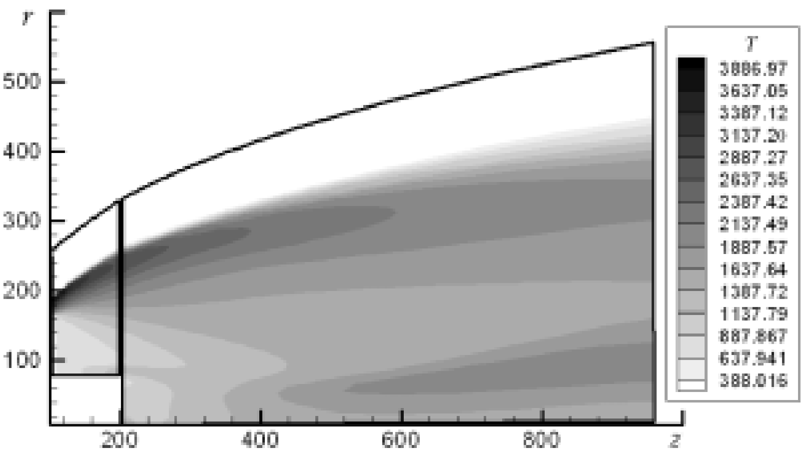

Fig. 4. Temperature distribution downstream of a space vehicle.

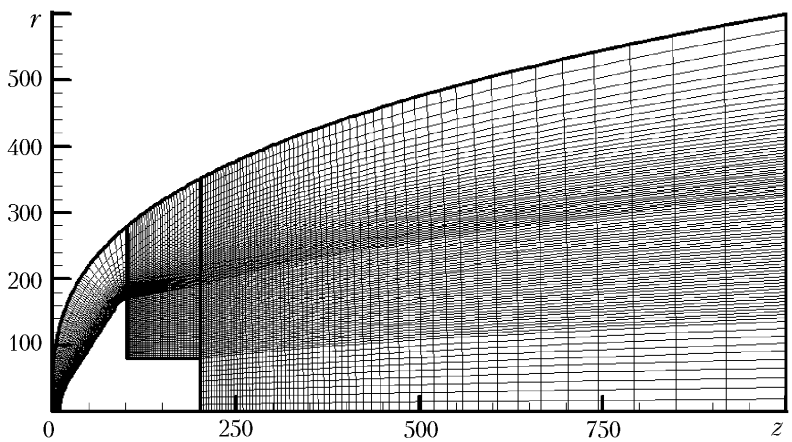

Fig. 5. Computational grid.

chart of the passage of cells in the order of their computation. A new chart of the sequence of involvement of the cells in the process of computation should be determined anew on change in the angular or three-dimensional grid.

Results of Computations. Using the method developed, calculations of the densities of radiation heat fluxes to the surface of the space vehicle selected by the European Space Agency as a base one were performed for a comparative estimation of various procedures of computation of the aerothermodynamics of space vehicles entering into atmosphere. It is assumed that the Mars atmosphere consists of 97% CO_2 and 3% N_2 (mass fractions). The optical range in which the main transport of heat by radiation occurs was selected is equal to 2.5–5.3 μm. This optical range is subsequently divided into the number of spectral groups equal to N_g, and in each group the radiation transfer equation is solved. The distributions of the temperature (Fig. 4) and mass concentrations of the components of the high-temperature mixture of gases near the surface of the space vehicle are taken from [10, 11]. The group absorption coefficients were calculated with the aid of the computer code ASTEROID [12]. The boundary conditions are formulated for an ideal black surface of the space vehicle at a temperature of 500 K; the gas at the boundary of the computational domain is cold and nonemitting; in the calculation of the fields of gasdynamic functions at the exit from the computational domain downstream along the coordinate lines, boundary conditions of the second kind were specified.

The gasdynamic calculations were performed on a sequence of curvilinear nonorthogonal structured grids. The initial, roughest, grid used in the radiation calculations is shown in Fig. 5. The location of each elementary computational cell in the Cartesian coordinate system can be characterized by the axial coordinate z and radius $r = \sqrt{x^2 + y^2}$, as well as by the angular coordinate $\theta \cup [0, 2\pi]$ that determines the turn of the r–z plane around the axis z. The tetrahedral grid was created from the indicated cylindrical grid by dividing the cylindrical cells into tetrahedrons (see Fig. 2).

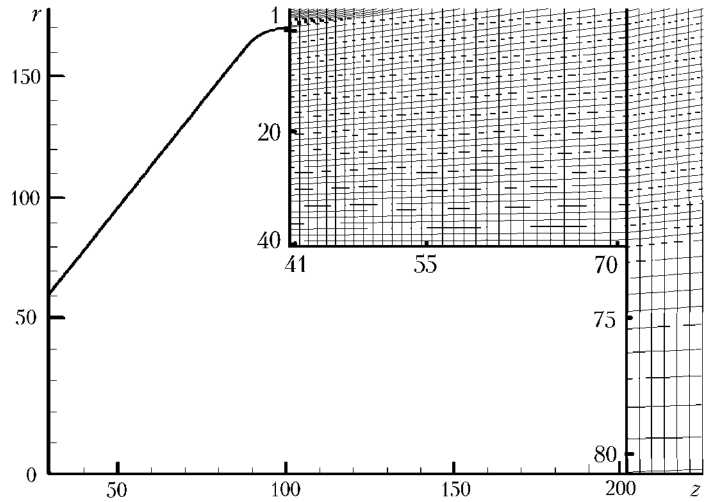

Fig. 6. Disposition of coordinate points J on the back surface of a space vehicle.

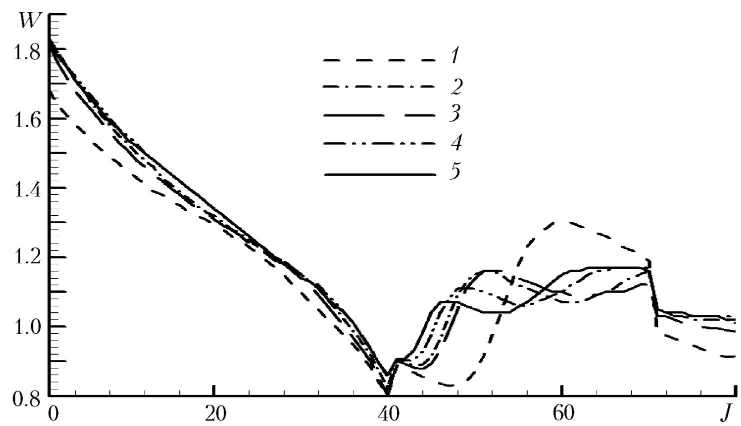

Fig. 7. Integral radiation flux along the back surface of a space vehicle at $N_\theta = 30$ and $N_g = 10$ depending on the order the quadrature S_N employed: 1–5) $S_N = S_6$, S_8, S_{10}, S_{12}, and S_{16}, respectively.

Figure 6 shows the order of location of the points J (numbering of the centers of the faces of cells) on the surface of the space vehicle in which radiation fluxes are calculated. The integral radiation fluxes on the surface of a space vehicle depend on the number of spectral ranges (N_g), the order of the quadrature employed (S_N), and also on the order of the angular splitting over the angle θ (N_θ). Methodical calculations for selecting optimal three-dimensional and angular grids, as well as for selecting the group approximation, were performed. Convergence of the solution is observed at the following parameters: $N_g = 200$, $N_\theta = 30$, and at the quadrature S_{16}. For example, in Fig. 7 the distribution of the density of integral radiation fluxes along the back surface of a space vehicle with the use of quadratures of different orders is shown. Calculations are presented for the 10-group optical model and $N_\theta = 30$.

To test the results obtained, similar calculations were carried out with the aid of the standard DOM [5, 6] in an orthogonal cylinder of radius $R = 600$ cm and height $Z = 900$ cm. The computational domain is divided into a nonuniform grid over r and z with 170 and 90 cells, respectively. The surface of the space vehicle was modeled by cells with a high absorption coefficient (10^5 cm^{-1}) and with a temperature of 500 K. The distributions of temperature (Fig. 4) and mass concentrations of the components of the high-temperature mixture of gases near the surface of the space vehicle on a cylindrical grid were calculated by the method of interpolation from a gas-dynamical grid (Fig. 5). Figure 8 demonstrates the distributions of integral radiation fluxes obtained for identical quadratures (S_{16}) and the numbers of spectral groups ($N_g = 200$) in calculations on structured and nonstructured grids.

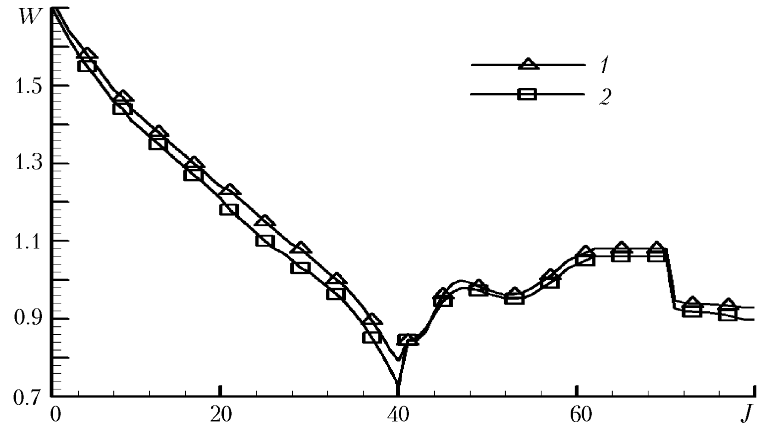

Fig. 8. Comparison of the integral fluxes obtained by the standard DOM along the back surface of a space vehicle at N_g = 200 and S_{16} quadrature (1) and by DOM on tetrahedral grids at N_g = 200, S_{16} quadrature, and at N_θ = 30 (2).

The advantage of the DOM is that this method allows one to rather simply find distributions of the radiation field characteristics on the entire computational domain rather than along the surface alone.

Conclusions. The DOM on tetrahedral grids made it possible to calculate the radiation fluxes incident on the back surface of a space vehicle of a complex shape in a group approximation. A comparative analysis for various orders of quadrature approximations, group approximations, and angular discretization is performed. The results obtained are compared with the standard DOM on a cylindrical grid. A comparison has shown the good accuracy and computational convergence of the method.

This work was carried out with support from the Russian Fundamental Research Foundation (grant No. 04-01-00237) and INTAS (grant No. 03-51-5204).

NOTATION

I^m, radiation intensity in the Ω_m direction, W/(cm^2·sr); I_b, blackbody radiation intensity, W/(cm^2·sr); J, number of the coordinate point on the surface of a descent module; **n**, unit normal vector; N_g, number of spectral groups; N_θ, number of points of division over the angle θ U [0, 2π]; r, z, θ, coordinates in the cylindrical coordinate system, cm, cm, rad; R, radius of a cylinder, cm; S_i, area of the ith face of a tetrahedron, cm^2; S_N, S quadrature of order N; T, temperature of the medium, K; V_p, volume of the cell numbered p, cm^3; W, flux to the surface, W/cm^2; x, y, z, coordinates in the Cartesian coordinate system, cm; Z, height of the cylinder, cm; χ, weight function; η_m, μ_m, and ξ_m, direction cosines of the discrete direction Ω_m; κ, absorption coefficient, cm^{-1}; λ, wavelength, μm; τ, optical length, cm; $\mathbf{\Omega}$, direction vector; ω_m, wight of the discrete direction $\mathbf{\Omega}_m$. Subscripts and superscripts: b, blackbody; i, number of the cell face; g, spectral group; m, number of discrete direction; p, center and number of a cell; Γ, surface bounding the computational domain.

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